

# Precalculus (IMRA)

Subject: Mathematics

Grade: 11

Expectations: 57

Breakouts: 269

## (a) Introduction.

1. The desire to achieve educational excellence is the driving force behind the Texas essential knowledge and skills for mathematics, guided by the college and career readiness standards. By embedding statistics, probability, and finance, while focusing on fluency and solid understanding, Texas will lead the way in mathematics education and prepare all Texas students for the challenges they will face in the 21st century.
2. The process standards describe ways in which students are expected to engage in the content. The placement of the process standards at the beginning of the knowledge and skills listed for each grade and course is intentional. The process standards weave the other knowledge and skills together so that students may be successful problem solvers and use mathematics efficiently and effectively in daily life. The process standards are integrated at every grade level and course. When possible, students will apply mathematics to problems arising in everyday life, society, and the workplace. Students will use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution. Students will select appropriate tools such as real objects, manipulatives, paper and pencil, and technology and techniques such as mental math, estimation, and number sense to solve problems. Students will effectively communicate mathematical ideas, reasoning, and their implications using multiple representations such as symbols, diagrams, graphs, and language. Students will use mathematical relationships to generate solutions and make connections and predictions. Students will analyze mathematical relationships to connect and communicate mathematical ideas. Students will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
3. Precalculus is the preparation for calculus. The course approaches topics from a function point of view, where appropriate, and is designed to strengthen and enhance conceptual understanding and mathematical reasoning used when modeling and solving mathematical and real-world problems. Students systematically work with functions and their multiple representations. The study of Precalculus deepens students' mathematical understanding and fluency with algebra and trigonometry and extends their ability to make connections and apply concepts and procedures at higher levels. Students investigate and explore mathematical ideas, develop multiple strategies for analyzing complex situations, and use technology to build understanding, make connections between representations, and provide support in solving problems.
4. Statements that contain the word "including" reference content that must be mastered, while those containing the phrase "such as" are intended as possible illustrative examples.

## (b) Knowledge and Skills Statements

- (1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
  - (A) apply mathematics to problems arising in everyday life, society, and the workplace;
    - (i) apply mathematics to problems arising in everyday life
    - (ii) apply mathematics to problems arising in society
    - (iii) apply mathematics to problems arising in the workplace

- (B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
- (i) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process
  - (ii) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the reasonableness of the solution
- (C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
- (i) select tools, including real objects as appropriate, to solve problems
  - (ii) select tools, including manipulatives as appropriate, to solve problems
  - (iii) select tools, including paper and pencil as appropriate, to solve problems
  - (iv) select tools, including technology as appropriate, to solve problems
  - (v) select techniques, including mental math as appropriate, to solve problems
  - (vi) select techniques including estimation as appropriate, to solve problems
  - (vii) select techniques, including number sense as appropriate, to solve problems
- (D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
- (i) communicate mathematical ideas using multiple representations, including symbols as appropriate
  - (ii) communicate mathematical ideas using multiple representations, including diagrams as appropriate
  - (iii) communicate mathematical ideas using multiple representations, including graphs as appropriate
  - (iv) communicate mathematical ideas using multiple representations, including language as appropriate
  - (v) communicate mathematical reasoning using multiple representations, including symbols as appropriate
  - (vi) communicate mathematical reasoning using multiple representations, including diagrams as appropriate
  - (vii) communicate mathematical reasoning using multiple representations, including graphs as appropriate
  - (viii) communicate mathematical reasoning using multiple representations, including language as appropriate
  - (ix) communicate [mathematical ideas'] implications using multiple representations, including symbols as appropriate
  - (x) communicate [mathematical ideas'] implications using multiple representations, including diagrams as appropriate
  - (xi) communicate [mathematical ideas'] implications using multiple representations, including graphs as appropriate
  - (xii) communicate [mathematical ideas'] implications using multiple representations, including language as appropriate
  - (xiii) communicate [mathematical reasoning's] implications using multiple representations, including symbols as appropriate

- (xiv) communicate [mathematical reasoning's] implications using multiple representations, including diagrams as appropriate
- (xv) communicate [mathematical reasoning's] implications using multiple representations, including graphs as appropriate
- (xvi) communicate [mathematical reasoning's] implications using multiple representations, including language as appropriate

(E) create and use representations to organize, record, and communicate mathematical ideas;

- (i) create representations to organize mathematical ideas
- (ii) create representations to record mathematical ideas
- (iii) create representations to communicate mathematical ideas
- (iv) use representations to organize mathematical ideas
- (v) use representations to record mathematical ideas
- (vi) use representations to communicate mathematical ideas

(F) analyze mathematical relationships to connect and communicate mathematical ideas; and

- (i) analyze mathematical relationships to connect mathematical ideas
- (ii) analyze mathematical relationships to communicate mathematical ideas

(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

- (i) display mathematical ideas using precise mathematical language in written or oral communication
- (ii) display mathematical arguments using precise mathematical language in written or oral communication
- (iii) explain mathematical ideas using precise mathematical language in written or oral communication
- (iv) explain mathematical arguments using precise mathematical language in written or oral communication
- (v) justify mathematical ideas using precise mathematical language in written or oral communication
- (vi) justify mathematical arguments using precise mathematical language in written or oral communication

(2) Functions. The student uses process standards in mathematics to explore, describe, and analyze the attributes of functions. The student makes connections between multiple representations of functions and algebraically constructs new functions. The student analyzes and uses functions to model real-world problems. The student is expected to:

(A) use the composition of two functions to model and solve real-world problems;

- (i) use the composition of two functions to model real-world problems
- (ii) use the composition of two functions to solve real-world problems

(B) demonstrate that function composition is not always commutative;

- (i) demonstrate that function composition is not always commutative

(C) represent a given function as a composite function of two or more functions;

- (i) represent a given function as a composite function of two or more functions

(D) describe symmetry of graphs of even and odd functions;

- (i) describe symmetry of graphs of even functions
  - (ii) describe symmetry of graphs of odd functions
- (E) determine an inverse function, when it exists, for a given function over its domain or a subset of its domain and represent the inverse using multiple representations;
- (i) determine an inverse function, when it exists, for a given function over its domain or a subset of its domain
  - (ii) represent the inverse using multiple representations
- (F) graph exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions;
- (i) graph exponential functions
  - (ii) graph logarithmic functions
  - (iii) graph rational functions
  - (iv) graph polynomial functions
  - (v) graph power functions
  - (vi) graph trigonometric functions
  - (vii) graph inverse trigonometric functions
  - (viii) graph piecewise defined functions, including step functions
- (G) graph functions, including exponential, logarithmic, sine, cosine, rational, polynomial, and power functions and their transformations, including  $af(x)$ ,  $f(x) + d$ ,  $f(x - c)$ ,  $f(bx)$  for specific values of  $a$ ,  $b$ ,  $c$ , and  $d$ , in mathematical and real-world problems;
- (i) graph functions, including exponential functions, in mathematical problems
  - (ii) graph functions, including exponential functions, in real-world problems
  - (iii) graph functions, including logarithmic functions, in mathematical problems
  - (iv) graph functions, including logarithmic functions, in real-world problems
  - (v) graph functions, including sine functions, in mathematical problems
  - (vi) graph functions, including sine functions, in real-world problems
  - (vii) graph functions, including cosine functions, in mathematical problems
  - (viii) graph functions, including cosine functions, in real-world problems
  - (ix) graph functions, including rational functions, in mathematical problems
  - (x) graph functions, including rational functions, in real-world problems
  - (xi) graph functions, including polynomial functions, in mathematical problems
  - (xii) graph functions, including polynomial functions, in real-world problems
  - (xiii) graph functions, including power functions, in mathematical problems
  - (xiv) graph functions, including power functions, in real-world problems

- (xv) graph functions, including exponential transformations, including  $af(x)$ , for specific values of  $a$ , in mathematical problems
- (xvi) graph functions, including exponential transformations, including  $af(x)$ , for specific values of  $a$ , in real-world problems
- (xvii) graph functions, including logarithmic transformations, including  $af(x)$ , for specific values of  $a$ , in mathematical problems
- (xviii) graph functions, including logarithmic transformations, including  $af(x)$ , for specific values of  $a$ , in real-world problems
- (xix) graph functions, including sine transformations, including  $af(x)$ , for specific values of  $a$ , in mathematical problems
- (xx) graph functions, including sine transformations, including  $af(x)$ , for specific values of  $a$ , in real-world problems
- (xxi) graph functions, including cosine transformations, including  $af(x)$ , for specific values of  $a$ , in mathematical problems
- (xxii) graph functions, including cosine transformations, including  $af(x)$ , for specific values of  $a$ , in real-world problems
- (xxiii) graph functions, including rational transformations, including  $af(x)$ , for specific values of  $a$ , in mathematical problems
- (xxiv) graph functions, including rational transformations, including  $af(x)$ , for specific values of  $a$ , in real-world problems
- (xxv) graph functions, including polynomial transformations, including  $af(x)$ , for specific values of  $a$ , in mathematical problems
- (xxvi) graph functions, including polynomial transformations, including  $af(x)$ , for specific values of  $a$ , in real-world problems
- (xxvii) graph functions, including power transformations, including  $af(x)$ , for specific values of  $a$ , in mathematical problems
- (xxviii) graph functions, including power transformations, including  $af(x)$ , for specific values of  $a$ , in real-world problems
- (xxix) graph functions, including exponential transformations, including  $f(x) + d$ , for specific values of  $d$ , in mathematical problems
- (xxx) graph functions, including exponential transformations, including  $f(x) + d$ , for specific values of  $d$ , in real-world problems
- (xxxi) graph functions, including logarithmic transformations, including  $f(x) + d$ , for specific values of  $d$ , in mathematical problems
- (xxxii) graph functions, including logarithmic transformations, including  $f(x) + d$ , for specific values of  $d$ , in real-world problems
- (xxxiii) graph functions, including sine transformations, including  $f(x) + d$ , for specific values of  $d$ , in mathematical problems
- (xxxiv) graph functions, including sine transformations, including  $f(x) + d$ , for specific values of  $d$ , in real-world problems

- (xxxv) graph functions, including cosine transformations, including  $f(x) + d$ , for specific values of  $d$ , in mathematical problems
- (xxxvi) graph functions, including cosine transformations, including  $f(x) + d$ , for specific values of  $d$ , in real-world problems
- (xxxvii) graph functions, including rational transformations, including  $f(x) + d$ , for specific values of  $d$ , in mathematical problems
- (xxxviii) graph functions, including rational transformations, including  $f(x) + d$ , for specific values of  $d$ , in real-world problems
- (xxxix) graph functions, including polynomial transformations, including  $f(x) + d$ , for specific values of  $d$ , in mathematical problems
  - (xl) graph functions, including polynomial transformations, including  $f(x) + d$ , for specific values of  $d$ , in real-world problems
  - (xli) graph functions, including power transformations, including  $f(x) + d$ , for specific values of  $d$ , in mathematical problems
  - (xlii) graph functions, including power transformations, including  $f(x) + d$ , for specific values of  $d$ , in real-world problems
  - (xliii) graph functions, including exponential transformations, including  $f(x - c)$ , for specific values of  $c$ , in mathematical problems
  - (xliv) graph functions, including exponential transformations, including  $f(x - c)$ , for specific values of  $c$ , in real-world problems
  - (xlv) graph functions, including logarithmic transformations, including  $f(x - c)$ , for specific values of  $c$ , in mathematical problems
  - (xlvi) graph functions, including logarithmic transformations, including  $f(x - c)$ , for specific values of  $c$ , in real-world problems
  - (xlvii) graph functions, including sine transformations, including  $f(x - c)$ , for specific values of  $c$ , in mathematical problems
  - (xlviii) graph functions, including sine transformations, including  $f(x - c)$ , for specific values of  $c$ , in real-world problems
  - (xlix) graph functions, including cosine transformations, including  $f(x - c)$ , for specific values of  $c$ , in mathematical problems
    - (l) graph functions, including cosine transformations, including  $f(x - c)$ , for specific values of  $c$ , in real-world problems
    - (li) graph functions, including rational transformations, including  $f(x - c)$ , for specific values of  $c$ , in mathematical problems
    - (lii) graph functions, including rational transformations, including  $f(x - c)$ , for specific values of  $c$ , in real-world problems
    - (liii) graph functions, including polynomial transformations, including  $f(x - c)$ , for specific values of  $c$ , in mathematical problems
    - (liv) graph functions, including polynomial transformations, including  $f(x - c)$ , for specific values of  $c$ , in real-world problems

- (lv) graph functions, including power transformations, including  $f(x - c)$ , for specific values of  $c$ , in mathematical problems
  - (lvi) graph functions, including power transformations, including  $f(x - c)$ , for specific values of  $c$ , in real-world problems
  - (lvii) graph functions, including exponential transformations, including  $f(bx)$  for specific values of  $b$ , in mathematical problems
  - (lviii) graph functions, including exponential transformations, including  $f(bx)$  for specific values of  $b$ , in real-world problems
  - (lix) graph functions, including logarithmic transformations, including  $f(bx)$  for specific values of  $b$ , in mathematical problems
  - (lx) graph functions, including logarithmic transformations, including  $f(bx)$  for specific values of  $b$ , in real-world problems
  - (lxi) graph functions, including sine transformations, including  $f(bx)$  for specific values of  $b$ , in mathematical problems
  - (lxii) graph functions, including sine transformations, including  $f(bx)$  for specific values of  $b$ , in real-world problems
  - (lxiii) graph functions, including cosine transformations, including  $f(bx)$  for specific values of  $b$ , in mathematical problems
  - (lxiv) graph functions, including cosine transformations, including  $f(bx)$  for specific values of  $b$ , in real-world problems
  - (lxv) graph functions, including rational transformations, including  $f(bx)$  for specific values of  $b$ , in mathematical problems
  - (lxvi) graph functions, including rational transformations, including  $f(bx)$  for specific values of  $b$ , in real-world problems
  - (lxvii) graph functions, including polynomial transformations, including  $f(bx)$  for specific values of  $b$ , in mathematical problems
  - (lxviii) graph functions, including polynomial transformations, including  $f(bx)$  for specific values of  $b$ , in real-world problems
  - (lxix) graph functions, including power transformations, including  $f(bx)$  for specific values of  $b$ , in mathematical problems
  - (lxx) graph functions, power transformations, including  $f(bx)$  for specific values of  $b$ , in real-world problems
- (H) graph  $\arcsin x$  and  $\arccos x$  and describe the limitations on the domain;
- (i) graph  $\arcsin x$
  - (ii) graph  $\arccos x$
  - (iii) describe the limitations on the domain
- (I) determine and analyze the key features of exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions such as domain, range, symmetry, relative maximum, relative minimum, zeros, asymptotes, and intervals over which the function is increasing or decreasing;
- (i) determine the key features of exponential functions

- (ii) determine the key features of logarithmic functions
  - (iii) determine the key features of rational functions
  - (iv) determine the key features of polynomial functions
  - (v) determine the key features of power functions
  - (vi) determine the key features of trigonometric functions
  - (vii) determine the key features of inverse trigonometric functions
  - (viii) determine the key features of piecewise defined functions, including step functions
  - (ix) analyze the key features of exponential functions
  - (x) analyze the key features of logarithmic functions
  - (xi) analyze the key features of rational functions
  - (xii) analyze the key features of polynomial functions
  - (xiii) analyze the key features of power functions
  - (xiv) analyze the key features of trigonometric functions
  - (xv) analyze the key features of inverse trigonometric functions
  - (xvi) analyze the key features of piecewise defined functions, including step functions
- (J) analyze and describe end behavior of functions, including exponential, logarithmic, rational, polynomial, and power functions, using infinity notation to communicate this characteristic in mathematical and real-world problems;
- (i) analyze end behavior of functions, including exponential functions, using infinity notation to communicate this characteristic in mathematical problems
  - (ii) analyze end behavior of functions, including exponential functions, using infinity notation to communicate this characteristic in real-world problems
  - (iii) analyze end behavior of functions, including logarithmic functions, using infinity notation to communicate this characteristic in mathematical problems
  - (iv) analyze end behavior of functions, including logarithmic functions, using infinity notation to communicate this characteristic in real-world problems
  - (v) analyze end behavior of functions, including rational functions, using infinity notation to communicate this characteristic in mathematical problems
  - (vi) analyze end behavior of functions, including rational functions, using infinity notation to communicate this characteristic in real-world problems
  - (vii) analyze end behavior of functions, including polynomial functions, using infinity notation to communicate this characteristic in mathematical problems
  - (viii) analyze end behavior of functions, including polynomial functions, using infinity notation to communicate this characteristic in real-world problems
  - (ix) analyze end behavior of functions, including power functions, using infinity notation to communicate this characteristic in mathematical problems



- (x) analyze end behavior of functions, including power functions, using infinity notation to communicate this characteristic in real-world problems
  - (xi) describe end behavior of functions, including exponential functions, using infinity notation to communicate this characteristic in mathematical problems
  - (xii) describe end behavior of functions, including exponential functions, using infinity notation to communicate this characteristic in real-world problems
  - (xiii) describe end behavior of functions, including logarithmic functions, using infinity notation to communicate this characteristic in mathematical problems
  - (xiv) describe end behavior of functions, including logarithmic functions, using infinity notation to communicate this characteristic in real-world problems
  - (xv) describe end behavior of functions, including rational functions, using infinity notation to communicate this characteristic in mathematical problems
  - (xvi) describe end behavior of functions, including rational functions, using infinity notation to communicate this characteristic in real-world problems
  - (xvii) describe end behavior of functions, including polynomial functions, using infinity notation to communicate this characteristic in mathematical problems
  - (xviii) describe end behavior of functions, including polynomial functions, using infinity notation to communicate this characteristic in real-world problems
  - (xix) describe end behavior of functions, including power functions, using infinity notation to communicate this characteristic in mathematical problems
  - (xx) describe end behavior of functions, including power functions, using infinity notation to communicate this characteristic in real-world problems
- (K) analyze characteristics of rational functions and the behavior of the function around the asymptotes, including horizontal, vertical, and oblique asymptotes;
- (i) analyze characteristics of rational functions
  - (ii) analyze the behavior of the function around the asymptotes, including horizontal asymptotes
  - (iii) analyze the behavior of the function around the asymptotes, including vertical asymptotes
  - (iv) analyze the behavior of the function around the asymptotes, including oblique asymptotes
- (L) determine various types of discontinuities in the interval  $(-\infty, \infty)$  as they relate to functions and explore the limitations of the graphing calculator as it relates to the behavior of the function around discontinuities;
- (i) determine various types of discontinuities in the interval  $(-\infty, \infty)$  as they relate to functions
  - (ii) explore the limitations of the graphing calculator as it relates to the behavior of the function around discontinuities
- (M) describe the left-sided behavior and the right-sided behavior of the graph of a function around discontinuities;
- (i) describe the left-sided behavior of the graph of a function around discontinuities
  - (ii) describe the right-sided behavior of the graph of a function around discontinuities
- (N) analyze situations modeled by functions, including exponential, logarithmic, rational, polynomial, and power functions, to solve real-world problems;

- (i) analyze situations modeled by functions, including exponential functions, to solve real-world problems
  - (ii) analyze situations modeled by functions, including logarithmic functions, to solve real-world problems
  - (iii) analyze situations modeled by functions, including rational functions, to solve real-world problems
  - (iv) analyze situations modeled by functions, including polynomial functions, to solve real-world problems
  - (v) analyze situations modeled by functions, including power functions, to solve real-world problems
- (O) develop and use a sinusoidal function that models a situation in mathematical and real-world problems; and
- (i) develop a sinusoidal function that models a situation in mathematical problems
  - (ii) develop a sinusoidal function that models a situation in real-world problems
  - (iii) use a sinusoidal function that models a situation in mathematical problems
  - (iv) use a sinusoidal function that models a situation in real-world problems
- (P) determine the values of the trigonometric functions at the special angles and relate them in mathematical and real-world problems.
- (i) determine the values of the trigonometric functions at the special angles
  - (ii) relate [values of trigonometric functions at special angles] in mathematical problems
  - (iii) relate [values of trigonometric functions at special angles] in real-world problems
- (3) Relations and geometric reasoning. The student uses the process standards in mathematics to model and make connections between algebraic and geometric relations. The student is expected to:
- (A) graph a set of parametric equations;
- (i) graph a set of parametric equations
- (B) convert parametric equations into rectangular relations and convert rectangular relations into parametric equations;
- (i) convert parametric equations into rectangular relations
  - (ii) convert rectangular relations into parametric equations
- (C) use parametric equations to model and solve mathematical and real-world problems;
- (i) use parametric equations to model mathematical problems
  - (ii) use parametric equations to model real-world problems
  - (iii) use parametric equations to solve mathematical problems
  - (iv) use parametric equations to solve real-world problems
- (D) graph points in the polar coordinate system and convert between rectangular coordinates and polar coordinates;
- (i) graph points in the polar coordinate system
  - (ii) convert between rectangular coordinates and polar coordinates
- (E) graph polar equations by plotting points and using technology;
- (i) graph polar equations by plotting points
  - (ii) graph polar equations by using technology

- (F) determine the conic section formed when a plane intersects a double-napped cone;
    - (i) determine the conic section formed when a plane intersects a double-napped cone
  - (G) make connections between the locus definition of conic sections and their equations in rectangular coordinates;
    - (i) make connections between the locus definition of conic sections and their equations in rectangular coordinates
  - (H) use the characteristics of an ellipse to write the equation of an ellipse with center  $(h, k)$ ; and
    - (i) use the characteristics of an ellipse to write the equation of an ellipse with center  $(h, k)$
  - (I) use the characteristics of a hyperbola to write the equation of a hyperbola with center  $(h, k)$ .
    - (i) use the characteristics of a hyperbola to write the equation of a hyperbola with center  $(h, k)$
- (4) Number and measure. The student uses process standards in mathematics to apply appropriate techniques, tools, and formulas to calculate measures in mathematical and real-world problems. The student is expected to:
- (A) determine the relationship between the unit circle and the definition of a periodic function to evaluate trigonometric functions in mathematical and real-world problems;
    - (i) determine the relationship between the unit circle and the definition of a periodic function to evaluate trigonometric functions in mathematical problems
    - (ii) determine the relationship between the unit circle and the definition of a periodic function to evaluate trigonometric functions in real-world problems
  - (B) describe the relationship between degree and radian measure on the unit circle;
    - (i) describe the relationship between degree and radian measure on the unit circle
  - (C) represent angles in radians or degrees based on the concept of rotation and find the measure of reference angles and angles in standard position;
    - (i) represent angles in radians or degrees based on the concept of rotation
    - (ii) find the measure of reference angles
    - (iii) find the measure of angles in standard position
  - (D) represent angles in radians or degrees based on the concept of rotation in mathematical and real-world problems, including linear and angular velocity;
    - (i) represent angles in radians or degrees based on the concept of rotation in mathematical problems, including linear velocity
    - (ii) represent angles in radians or degrees based on the concept of rotation in real-world problems, including linear velocity
    - (iii) represent angles in radians or degrees based on the concept of rotation in mathematical problems, including angular velocity
    - (iv) represent angles in radians or degrees based on the concept of rotation in real-world problems, including angular velocity
  - (E) determine the value of trigonometric ratios of angles and solve problems involving trigonometric ratios in mathematical and real-world problems;
    - (i) determine the value of trigonometric ratios of angles

- (ii) solve problems involving trigonometric ratios in mathematical problems
  - (iii) solve problems involving trigonometric ratios in real-world problems
  - (F) use trigonometry in mathematical and real-world problems, including directional bearing;
    - (i) use trigonometry in mathematical, including directional bearing
    - (ii) use trigonometry in real-world problems, including directional bearing
  - (G) use the Law of Sines in mathematical and real-world problems;
    - (i) use the Law of Sines in mathematical problems
    - (ii) use the Law of Sines in real-world problems
  - (H) use the Law of Cosines in mathematical and real-world problems;
    - (i) use the Law of Cosines in mathematical problems
    - (ii) use the Law of Cosines in real-world problems
  - (I) use vectors to model situations involving magnitude and direction;
    - (i) use vectors to model situations involving magnitude
    - (ii) use vectors to model situations involving direction
  - (J) represent the addition of vectors and the multiplication of a vector by a scalar geometrically and symbolically; and
    - (i) represent the addition of vectors geometrically
    - (ii) represent the addition of vectors symbolically
    - (iii) represent the multiplication of a vector by a scalar geometrically
    - (iv) represent the multiplication of a vector by a scalar symbolically
  - (K) apply vector addition and multiplication of a vector by a scalar in mathematical and real-world problems.
    - (i) apply vector addition in mathematical problems
    - (ii) apply vector addition in real-world problems
    - (iii) apply multiplication of a vector by a scalar in mathematical problems
    - (iv) apply multiplication of a vector by a scalar in real-world problems
- (5) Algebraic reasoning. The student uses process standards in mathematics to evaluate expressions, describe patterns, formulate models, and solve equations and inequalities using properties, procedures, or algorithms. The student is expected to:
- (A) evaluate finite sums and geometric series, when possible, written in sigma notation;
    - (i) evaluate finite sums written in sigma notation
    - (ii) evaluate geometric series, when possible, written in sigma notation
  - (B) represent arithmetic sequences and geometric sequences using recursive formulas;
    - (i) represent arithmetic sequences using recursive formulas
    - (ii) represent geometric sequences using recursive formulas
  - (C) calculate the  $n^{\text{th}}$  term and the  $n^{\text{th}}$  partial sum of an arithmetic series in mathematical and real-world problems;

- (i) calculate the  $n^{\text{th}}$  term of an arithmetic series in mathematical problems
  - (ii) calculate the  $n^{\text{th}}$  term of an arithmetic series in real-world problems
  - (iii) calculate the  $n^{\text{th}}$  partial sum of an arithmetic series in mathematical problems
  - (iv) calculate the  $n^{\text{th}}$  partial sum of an arithmetic series in real-world problems
- (D) represent arithmetic series and geometric series using sigma notation;
- (i) represent arithmetic series using sigma notation
  - (ii) represent geometric series using sigma notation
- (E) calculate the  $n^{\text{th}}$  term of a geometric series, the  $n^{\text{th}}$  partial sum of a geometric series, and sum of an infinite geometric series when it exists;
- (i) calculate the  $n^{\text{th}}$  term of a geometric series
  - (ii) calculate the  $n^{\text{th}}$  partial sum of a geometric series
  - (iii) calculate the sum of an infinite geometric series when it exists
- (F) apply the Binomial Theorem for the expansion of  $(a + b)^n$  in powers of  $a$  and  $b$  for a positive integer  $n$ , where  $a$  and  $b$  are any numbers;
- (i) apply the Binomial Theorem for the expansion of  $(a + b)^n$  in powers of  $a$  and  $b$  for a positive integer  $n$ , where  $a$  and  $b$  are any numbers
- (G) use the properties of logarithms to evaluate or transform logarithmic expressions;
- (i) use the properties of logarithms to evaluate or transform logarithmic expressions
- (H) generate and solve logarithmic equations in mathematical and real-world problems;
- (i) generate logarithmic equations in mathematical problems
  - (ii) generate logarithmic equations in real-world problems
  - (iii) solve logarithmic equations in mathematical problems
  - (iv) solve logarithmic equations in real-world problems
- (I) generate and solve exponential equations in mathematical and real-world problems;
- (i) generate exponential equations in mathematical problems
  - (ii) generate exponential equations in real-world problems
  - (iii) solve exponential equations in mathematical problems
  - (iv) solve exponential equations in real-world problems
- (J) solve polynomial equations with real coefficients by applying a variety of techniques in mathematical and real-world problems;
- (i) solve polynomial equations with real coefficients by applying a variety of techniques in mathematical problems
  - (ii) solve polynomial equations with real coefficients by applying a variety of techniques in real-world problems
- (K) solve polynomial inequalities with real coefficients by applying a variety of techniques and write the solution set of the polynomial inequality in interval notation in mathematical and real-world problems;

- (i) solve polynomial inequalities with real coefficients by applying a variety of techniques in mathematical problems
  - (ii) solve polynomial inequalities with real coefficients by applying a variety of techniques in real-world problems
  - (iii) write the solution set of the polynomial inequality in interval notation in mathematical problems
  - (iv) write the solution set of the polynomial inequality in interval notation in real-world problems
- (L) solve rational inequalities with real coefficients by applying a variety of techniques and write the solution set of the rational inequality in interval notation in mathematical and real-world problems;
- (i) solve rational inequalities with real coefficients by applying a variety of techniques
  - (ii) write the solution set of the rational inequality in interval notation in mathematical problems
  - (iii) write the solution set of the rational inequality in interval notation in real-world problems
- (M) use trigonometric identities such as reciprocal, quotient, Pythagorean, cofunctions, even/odd, and sum and difference identities for cosine and sine to simplify trigonometric expressions; and
- (i) use trigonometric identities for cosine to simplify trigonometric expressions
  - (ii) use trigonometric identities for sine to simplify trigonometric expressions
- (N) generate and solve trigonometric equations in mathematical and real-world problems.
- (i) generate trigonometric equations in mathematical problems
  - (ii) generate trigonometric equations in real-world problems
  - (iii) solve trigonometric equations in mathematical problems
  - (iv) solve trigonometric equations in real-world problems